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INTERACTION OF AN EXTERNAL DISTURBANCE WITH TURBULENT FLOW

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Previous calculations [1] and a critical analysis of the interpretation of some experimental data [2, 3] are verified and refined. A model is proposed that directly takes into account in the motion equations terms describing the interaction of the disturbance with turbulent oscillations. The advantages of such an approach in comparison with the use of turbulent viscosity models are demonstrated.

Interest in the stability of turbulent flows has recently grown in connection with attempts to predict the averaged characteristics of turbulent flow based on stability properties [4-7]. The stability problem as of now has been solved only in a quasilaminar approximation, in which the interaction of the disturbance with fluctuations is not taken into account [5]. This is due to the absence of experimental data that would permit any given model describing such interaction to be accepted. A series of works by Reynolds and Hussain [1-3], in which original experiments and the first calculations using models taking into account the interaction of a weak nonrandom signal from the turbulence for channel flow were performed, appeared in 1970-1972.

A periodic perturbation (vibrating streaks near walls) was introduced in a given section of the channel and its downstream propagation was studied. A weak, nonrandom signal consisting of about 4% of the turbulent velocity fluctuations was isolated. Experiments were carried out for four frequencies with a Reynolds number (Re = 13,800) calculated according to the channel half-width and maximal velocity [2].

A spatial stability problem for turbulent flow to a linear approximation arose as a result of this experiment. The exponential nature of signal attenuation was indicated by the validity of the linear approximation [2-3].

The disturbance equations have the form

$$\frac{\partial \langle v_i \rangle}{\partial x_j} + \frac{\partial (U_j \langle v_i \rangle + U_i \langle v_j \rangle)}{\partial x_j} = -\frac{\partial \langle p \rangle}{\partial x_i} + \frac{1}{\mathrm{Re}} \frac{\partial^2 \langle v_i \rangle}{\partial x_j \partial x_j} - \frac{\partial}{\partial x_j} \langle v_i v_j^m + v_i^m v_j \rangle; \quad \frac{\partial \langle v_j \rangle}{\partial x_j} = 0, \tag{1}$$

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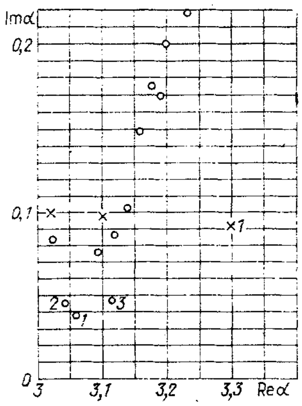

Fig. 1

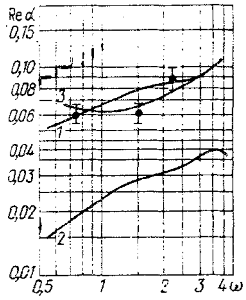

Fig. 2

where v_i and p are the velocity and pressure disturbances, v_i^m is the fluctuating velocity in undisturbed flow, and U_i is averaged velocity. Brackets denote the ensemble average.

The last term in the right side of Eq. (1) describes the interaction of the disturbance with the turbulent fluctuations. It is discarded in the quasilaminar approximation. This term was taken into account in models proposed in [1] by means of the effective turbulent viscosity. The solution was found in the form

$$\langle v_i \rangle = u_i(y) e^{i(\alpha x - \omega t)}, \tag{2}$$

where x and y are coordinates along and across the channel, ω is a given frequency, and α is a complex eigenvalue. If the last term of Eq. (1) is represented in this form,

$$\langle v_i v_j^m + v_i^m v_j \rangle = \tau_{ij}(y) e^{i(\alpha x - \omega t)},$$

where τ_{ij} is the complex amplitude of the fluctuations of disturbances of Reynolds stresses, an Orr–Sommerfeld type equation [1] is obtained for the complex fluctuation amplitudes,

$$\alpha(U_x - c)(u_y'' - \alpha^2 u_y) - \alpha U_x'' u_y = -\frac{i}{\mathrm{Re}}(u_y^{\mathrm{IV}} - 2\alpha^2 u_y'' + \alpha^4 u_y) + i\alpha^2(\tau_{xx}' - \tau_{yy}') + \alpha(\tau_{xy}'' + \alpha^2 \tau_{xy}).$$

The last two terms were discarded in previous [1] calculations and the number 1/Re was replaced as compensation by $(1/\mathrm{Re}) + (1/\mathrm{Re_m})$, where $\mathrm{Re_m} = (\nu/\varepsilon)$ is the turbulent Reynolds number calculated relative to turbulent viscosity ε (ν is molecular kinematic viscosity). A comparison of experimental data to results of previously performed [1] calculations demonstrated that the attenuation decrements calculated in terms of the quasilaminar model significantly exceed the experimental values as well as those calculated using models with turbulent viscosity. The latter result is somewhat unexpected, since it had been anticipated that a calculation for interaction of a disturbance with fluctuations would lead to stabilization. These calculations were therefore checked.

Several first eigenvalues (numbered in increasing order of decrement) calculated using the quasilaminar model by means of the differential pivotal compensation method [8] for $\omega = 3$ and Re = 13,800 are represented in Fig. 1 by circles. Figure 2 (curve 2) depicts the dependence of attenuation damping on frequency for the first mode.

The eigenvalues can be divided into two classes corresponding to near-axial and boundary modes. Boundary modes are characterized by the fact that their phase velocity decreases and the critical layer shifts towards the wall as frequency increases, whereas phase velocity increases with frequency and the critical layer shifts towards the axis in near-axial modes. It was clarified that not the first mode, but the mode with number greater than 30 that turned out to be the first of the boundary modes had been calculated in [1] using a quasilaminar model.

Good agreement with experiment using the quasilaminar approximation can be expected in two cases, namely, for boundary modes under locality conditions [9], and for short waves for a relatively weak degree of turbulence. In both cases, the last term in Eqs. (1) becomes insignificant. In fact, if we add the last term in the right side of Eq. (1) and the last term in the left side, we obtain

$$\left\langle \frac{\partial}{\partial x_j}\left[(U_j + v_j^m) v_i + (U_i + v_i^m) v_j\right]\right\rangle$$

or, after differentiating using the continuity equation,

$$\left\langle (U_j + v_j^m)\frac{\partial v_i}{\partial x_j} + v_j \frac{\partial (U_i + v_i^m)}{\partial x_j} \right\rangle. \tag{3}$$

The first term in Eq. (3) is negligible in comparison with the second term, which contains the velocity gradients, near the wall, where the velocities are low. When weak turbulence occurs in this region, $|\text{grad } U_x| \gg |\text{grad } v_i^m|$, so that the influence of turbulence in insignificant and boundary modes under locality conditions [9] ($\text{Re}\alpha \geqslant 1.5$) will be reasonably described by the quasilaminar model. The first term is much greater than the second in the case of short waves near the axis, but v^m is negligible in comparison with U for weak turbulence in it, and the quasilaminar model is again suitable. In the case of long waves the second term in Eq. (3) becomes substantial, and the quasilaminar approximation cannot be carried out, which leads us to conclude that it is necessary to use the first boundary mode in order to compare the calculation using the quasilaminar model to experiment. But an analysis of previous [2, 3] data has shown that only near-axial modes are present in an experiment on the stabilization segment and, consequently, a comparison must be carried out in terms of near-axial modes. Here, the quasilaminar model yields substantially understated attenuation decrements, since $|\text{grad } v_i^m| \gg |\text{grad } U_x|$ near the axis, and the influence of the last term in Eq. (3) again becomes substantial. If this influence is taken into account, there is no meaning to considering the cross-sectionally variable turbulent viscosity and it can be replaced by a constant independent of frequency under locality conditions for near-axial modes, in which $\text{Re}\,\alpha \geqslant 0.2$, by introducing, as was done in [1], the effective turbulent viscosity [9].

A model of constant turbulent viscosity was designed in [1] for $\varepsilon/\nu = 40$. The first mode was near-axial. Results of these calculations were confirmed by us, but the comparison to experimental data as well as the interpretation of the latter was not satisfactory. Since some authors [2, 3] could not isolate a unique mode, the decrements obtained as the cross-sectionally mean decrements or relative to some maximum more likely reflect interaction of modes than attenuation.

The experiment demonstrated that near-axial modes are attenuated least. The first mode, calculated using the turbulent viscosity model, also turned out to be near-axial. It would therefore be of interest for the comparison to isolate the attenuation decrements near the axis. An analysis of the curves presented in [3] demonstrated that attenuation decrements for $y/\delta = 0.1$ (δ is the channel half-width and the distance y is counted off from the axis) are reasonably described by a constant turbulent viscosity model $\varepsilon/\nu = 40$ for ω between 0.75 and 2.25 (cf. Fig. 2, curve 1). When $\omega = 3$ an increment occurs at the point $y/\delta = 0.1$. That is, the different modes overlap too strongly and the data for $\omega = 3$ should not be taken into account. It remains unclear why it is necessary to set $\varepsilon/\nu = 40$ since $\varepsilon/\nu = 80$ under these conditions, according to some data [10].

The signal attenuation problem for turbulent flow is complicated by the fact that a disturbance implies a staggered process. The energy of a signal with a preassigned frequency is transmitted to other frequencies. An exact solution requires that we find each of the resulting harmonics and determine the staggered process itself, which as yet cannot be done. Our problem is to study the behavior of a signal of the initial frequency. The influence of turbulence on it will only effectively be taken into account. If this is possible, the given influence will be such that the motion equations permit a harmonic solution. This implies first, that the simulated term be linear relative to disturbance velocity,

$$\langle v_i v_j^m + v_i^m v_j \rangle = v_i w_j + v_j w_i, \tag{4}$$

and, secondly, the vector **w**, which has the meaning of mean velocity and which must be determined only by the averaged characteristics of turbulent flow, must be independent of the homogeneous variables; in general, **w** will depend on all the momenta, though in constructing it we will limit ourselves only to the first and second momenta, as a first trial step; the averaged velocity and Reynolds stress tensor are invariant with it.

A number of requirements must be imposed on the model of the last term in Eq. (1)

$$\frac{\partial}{\partial x_j} \langle v_i v_j^m + v_i^m v_j \rangle. \tag{5}$$

These include: 1) linearity relative to disturbance velocity; 2) linearity relative to the intensity of turbulent fluctuations; 3) nearness to the simulated object in order of magnitude and preservation of the decreasing order towards the wall; 4) preservation of the order of differentiation and divergence; 5) locality; and 6) preservation of symmetry properties.

We note that the substitution of Eq. (4) in (1) is equivalent to the replacement of U_i by $U_i + w_i$, i.e., the vector **w** plays the role of a given auxiliary velocity. It will be independent of the average velocity and can therefore be determined only by the Reynolds stress tensor component. The vector **U** may turn out to have only nominal influence. The vector w will be near in order of magnitude to the standard deviation of the turbulent velocity fluctuations.

Only two invariant combinations of our averaged magnitudes, giving the necessary order of magnitude for **w**, can be indicated:

$$w_i = \gamma \frac{\langle v_i^m v_j^m \rangle}{\sqrt{\langle v_k^m v_k^m \rangle}} \frac{U_j}{|U|}; \tag{6}$$

$$w_i = \gamma \sqrt{\langle v_i^m v_j^m \rangle} \frac{U_j}{|U|}. \tag{7}$$

Here γ is a numerical coefficient on the order of unity.

The requirements 1-5 are satisfied when each of these combinations is substituted in Eq. (5). Requirement 6 is satisfied only by the combination (6), since (7) does not satisfy the symmetry condition.

The numerical coefficient γ in Eq. (6) will be determined from the following concepts. Since turbulence is nearly homogeneous and isotropic in the flow core, $\langle (v_x^m)^2 \rangle \approx \langle (v_y^m)^2 \rangle \approx \langle (v_z^m)^2 \rangle$, we will require that $w_x \approx \sqrt{\langle (v_x^m)^2 \rangle}$ so that $\gamma \simeq \sqrt{3}$ on the axis.

Substituting Eq. (4) in (1) and finding the solution in the form (2), we find the following equation for the complex fluctuation amplitudes:

$$\alpha (U_x + w_x - c) (u_y'' - \alpha^2 u_y) - \alpha (U_x'' + w_x'') u_y =$$
$$-\frac{i}{\mathrm{Re}} (u_y^{\mathrm{IV}} - 2\alpha^2 u_y'' + \alpha^4 u_y) + i \left[(w_y u_y')'' - \alpha^2 (w_y u_y)' - \alpha^2 (w_y' u_y) \right]. \tag{8}$$

Disturbances symmetric in y were calculated for comparison with experiment, so that the boundary conditions have the form

$$\begin{aligned} u_y = u_y' &= 0 \quad \text{on the wall,} \\ u_y' = u_y''' &= 0 \quad \text{on the axis.} \end{aligned} \tag{9}$$

The calculations were carried out for Eqs. (8) and (9), taking into account Eq. (6) when $\gamma = 3^{1/2}$. Averaged characteristics were taken from experimental data [2, 11]. The equation was solved by the differential pivotal condensation method [8].

It is evident from Fig. 1 (the first eigenvalues for $\omega = 3$ are plotted by crosses) that the model predicts the existence of several modes with rather similar attenuation decrements and similar wave numbers. The first modes turn out to be near-axial. The model reasonably describes attenuation near the axis for the experimentally studied range of frequencies (cf. Fig. 2, curve 3). Attenuation decrements for one mode in the experiment were not distinguished, so that the comparison was carried out in terms of data in the near-axial zone.

We note that the model and modelled objects are near in order of magnitude, so that the influence of a new term occurs only wherever the quasilaminar approximation is inapplicable. This is an advantage relative to turbulent viscosity models in which it is necessary to introduce a dependence on frequency. In particular, the effective turbulent viscosity ε will vanish for short waves in which the disturbance loses energy, preferably by a viscous mechanism. An attempt has been undertaken [12] to provide a basis for turbulent viscosity models and to explain the corresponding dependence on frequency, which will be complex and artificial, since the introduction of turbulent viscosity in place of the last term of Eq. (1) will not reflect the nature of the modelled object, but simply replace one mechanism for energy transfer by another.

Unfortunately, there are as yet too few experimental data on which definitive conclusions could be drawn. It is, however, evident that direct simulation of a term describing the interaction of a disturbance with fluctuations, reflecting a number of important properties of the simulated object, will allow us to describe the phenomenon relatively simply without introducing large coefficients that depend on frequency in a complex manner.

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DEVELOPMENT OF QUASIHARMONIC MOTIONS OF A GAS-STREAMLINED LIQUID FILM

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UDC 532.62+532.592+594

Quasiharmonic wave motions of a thin liquid film flowing in a vertical plane due to gravitational force, capillary forces, and a tangential stress acting on the film–gas boundary are considered. The region of existence and spectral characteristics of the quasiharmonic wave solutions in different film-motion regimes (cocurrent and countercurrent) are found.

§1. Let us consider the motion of a thin film of a viscous liquid flowing in a vertical plane, under the influence of gravitational and capillary forces and of stresses arising on the film surface as it is streamlined by gas. As in [1, 2], we replace the closed combined motion problem of the gas and liquid (in the film) by motion problems of a single film only. The effect of the gas on the film in the problem thus reduced is described by specifying the tangential (and normal) stresses on the gas–film boundary. The exact form of these stresses is unknown within the context of this procedure. We assume, as in [1, 2], that the tangential stress on the film

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